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S-brane solutions in gauged and ungauged supergravities

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Abstract

We construct time dependent S-brane solutions in gauged and ungauged supergravities in various dimensions. The supergravity solutions we find are all special cases of solutions in gauged supergravities with symmetric potentials. We discuss some properties of these solutions and their relation to topological black holes in anti-de Sitter spaces.

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1 Introduction

Time dependent solutions in string theory and supergravities have been of great interest recently. Classical solutions of supergravity theories can be the starting point to analyze questions of stability, particle creation and singularity resolution in string theory. It is important to understand these issues better in order to apply string theory to cosmology.

A particular class of such solutions are called S-branes (for 'spacelike' branes) [1]. S-branes describe a shell of radiation moving in from infinity and forming an unstable brane which subsequently decays. Since the brane only exists for a finite time it is localized in time and is therefore spacelike.

A concrete realization of this idea was developed by Sen in terms of open string tachyon condensation in [2–4]. Recently various supergravity S-brane solutions were found in [5–13] (For related earlier work see [14–17]). The ansatz for an S-p brane supergravity solution in d dimensions has $ISO(p+1) \times SO(d-p-2,1)$ symmetry. The first factor corresponds to the symmetry of the (spacelike) worldvolume and the second factor to the symmetry of the transverse lightcone. In the following we will only consider S-0 branes with one dimensional worldvolume.

Note that all of these solutions (as well as the ones we find) have singularities. However nonsingular S-brane solutions have been obtained recently in [18–21].

In this note we find S-brane solutions in gauged and ungauged supergravities. In section 2 we review and discuss how to obtain S-brane solutions by analytically continuing black hole solutions. In the case of black holes in AdS-space we point out a relation of S-brane solutions to topological AdS black holes [22].

In section 3 the equations of motion for an S-brane in a theory with nontrivial scalars and gauge fields in arbitrary dimensions are presented. In section 4 we present S-brane solutions in a gauged supergravity where the scalars lie in a coset $SL(n,R)/SO(n,R)$ and the potential is given by a symmetric function of the scalars. The solutions are special because the metric, scalars and gauge fields are all expressed in terms of harmonic functions. In section 5 to 7 we find simple solutions of some particular (un)gauged supergravities in dimensions $d = 5, 4$ and 7 respectively. These solutions can be obtained from the solution presented in section 4 by an identification of scalar and gauge fields.

All the solutions we find have similar properties: The reality of the solution imposes

constraints on the charges of the solution and the 'nonextremality' parameter. In the case of gauged supergravity the S-brane solutions are asymptotically AdS and are related to topological black holes in AdS.

2 Black hole solutions and S-branes in flat space and AdS

Many S-brane and related time dependent solutions can be obtained by analytic continuation of known static solutions. In this section we will review this method using some simple examples which have been previously discussed in the literature [10–12]. We start with the metric for the four dimensional Schwarzschild black hole.

$$ds^2 = -(1 - \frac{m}{r})dt^2 + \frac{1}{1 - \frac{m}{r}}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{2.1}$$

A time dependent solution can be obtained from (2.1) by replacing

$$r \to i\tau, \quad t \to ix, \quad m \to im, \quad \theta \to i\theta, \tag{2.2}$$

which gives

$$ds^2 = -\frac{1}{1 - \frac{m}{\tau}}d\tau^2 + (1 - \frac{m}{\tau})dx^2 + \tau^2(d\theta^2 + \sinh^2\theta d\phi^2). \tag{2.3}$$

The continuation turns the metric on the two sphere S_2 into the metric on H_2, the Poincaré plane (or Euclidean AdS_2). There is a coordinate singularity at $t = m$ which defines a horizon. Continuing the coordinates, one gets a static metric and a timelike curvature singularity at $t = 0$. The Penrose diagram of this spacetime is the one of the Schwarzschild black hole which is rotated by 90 degrees.

Another example is given by the Reissner-Nordström solution of Einstein-Maxwell theory

$$\begin{aligned} ds^2 &= -(1 - \frac{2m}{r} + \frac{q^2}{r^2})dt^2 + (1 - \frac{2m}{r} + \frac{q^2}{r^2})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \\ F_{rt} &= \frac{q}{r^2}. \end{aligned} \tag{2.4}$$

The analytic continuation is given by

$$r \to i\tau, \quad t \to ix, \quad m \to \pm im, \quad q \to q, \quad \theta \to i\theta \tag{2.5}$$

and produces the metric

$$ds^2 = -(1 \mp \frac{2m}{\tau} - \frac{q^2}{\tau^2})^{-1} d\tau^2 + (1 \mp \frac{2m}{\tau} - \frac{q^2}{\tau^2}) dx^2 + \tau^2 (d\theta^2 + \sinh^2\theta d\phi^2),$$
$$F_{\tau x} = \frac{q}{\tau^2}. \tag{2.6}$$

An extremal black hole obeys $m = \pm q$. In this case analytic continuation (2.5) produces a complex field-strength and the solution obtained is therefore unphysical. This indicates a generic feature that the S-brane solutions obtained by analytic continuation are always nonsupersymmetric, as expected from spacetimes without timelike or null Killing vectors.

The next example we discuss is a black hole in AdS_5, whose metric is given by

$$ds^2 = -(1 - \frac{m}{r^2} + r^2) dt^2 + \frac{1}{1 - \frac{m}{r^2} + r^2} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\Omega_2^2), \tag{2.7}$$

where $d\Omega_2^2$ is the metric on the unit two sphere. As before one can obtain a real time dependent solution from the above metric by performing the analytic continuation [1]

$$r \to i\tau, \quad t \to ix, \quad \theta \to i\theta, \tag{2.8}$$

which should be associated with an S-brane in AdS. The analytically continued metric is

$$ds^2 = (1 + \frac{m}{\tau^2} - \tau^2) dx^2 - \frac{1}{1 + \frac{m}{\tau^2} - \tau^2} d\tau^2 + \tau^2 (d\theta^2 + \sinh^2\theta d\Omega_2^2). \tag{2.9}$$

There are coordinate singularities at $\tau_\pm^2 = \frac{1}{2}(1 \pm \sqrt{1+4m})$. For $0 > m > -1/4$ there are two horizons $\tau_\pm$, whereas for $m > 0$ there is only one horizon at τ_+. For $\tau > \tau_+$ the metric becomes static and one gets

$$ds^2 = -(-1 - \frac{m}{\tau^2} + \tau^2) dx^2 + \frac{1}{-1 - \frac{m}{\tau^2} + \tau^2} d\tau^2 + \tau^2 dH_3^2. \tag{2.10}$$

This metric is the ‘topological’ black hole solution [26–28], i.e. a black hole in AdS with a hyperbolic (or quotients thereof) horizon. In contrast to the asymptotically flat case, in AdS the analytic continuation (2.8) of a black hole with a spherical horizon does not produce a new time dependent background but instead produces the inside of a black hole with a hyperbolic horizon. Since the horizon is non-compact, this solution might have a cosmological interpretation.

[1] A different analytic continuation was given in [23] relating the AdS BH to fluxbranes [24, 25]

3 General Considerations

In this section we consider theories of d-dimensional gravity coupled to gauge and scalar fields described by the general Lagrangian

$$e^{-1}\mathcal{L}_d = R - \frac{1}{2}\mathcal{M}_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - \frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{\mu\nu\,J} - l^2\mathcal{P}. \tag{3.1}$$

Here $\mathcal{M}_{ij}$, G_{IJ} and $\mathcal{P}$ are functions of the scalar fields ϕ^i. The Einstein gravitational equations derived from (3.1) are given by

$$R_{\mu\nu} = \frac{1}{2}\mathcal{M}_{ij}\partial_\mu\phi^i\partial_\nu\phi^j + \frac{1}{2}G_{IJ}\left(F^I_{\mu\lambda}F_\nu{}^{\lambda\,J} - \frac{1}{2(d-2)}g_{\mu\nu}F^I_{\rho\sigma}F^{\rho\sigma\,J}\right) + \frac{g_{\mu\nu}}{d-2}l^2\mathcal{P}. \tag{3.2}$$

The variation of (3.1) with respect to the scalar and gauge fields gives, respectively, the scalar equation of motion

$$\frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}g^{\mu\nu}\mathcal{M}_{ij}\partial_\nu\phi^j) - \frac{1}{2}\partial_i\mathcal{M}_{kl}\partial_\mu\phi^k\partial^\mu\phi^l - \frac{1}{4}\partial_i G_{JK}F^J_{\mu\nu}F^{K\mu\nu} - l^2\partial_i\mathcal{P} = 0 \tag{3.3}$$

and the equations of motion for the abelian gauge fields

$$D_\mu(G_{IJ}F^{\mu\nu\,J}) = 0. \tag{3.4}$$

For a solution we take the following ansatz for the metric

$$ds^2 = -e^{2A(t)}dt^2 + e^{2B(t)}dx^2 + e^{2C(t)}ds^2_{d-2,k} \tag{3.5}$$

where $ds^2_{d-2,k} = \bar{g}_{ab}dx^a dx^b$ $(a,b = 1,..,d-2)$ and the Ricci tensor for the metric $\bar{g}_{ab}$ is given by $\bar{R}_{ab} = k(d-3)\bar{g}_{ab}$, with $k = 1,-1,0$. We also take $A^I_x(t)$ to be the only non-vanishing component of the vector potentials. The non-vanishing components of the Ricci tensor are then given by

$$\begin{aligned}
R_{xx} &= e^{2B-2A}(B'' + B'^2 - A'B' + (d-2)B'C'),\\
R_{tt} &= -(B'' + B'^2 - A'B') - (d-2)\left(C'' + C'^2 - A'C'\right),\\
R_{ab} &= \left[e^{2C-2A}(C'' - A'C' + B'C' + (d-2)C'^2) + k(d-3)\right]\bar{g}_{ab}.
\end{aligned} \tag{3.6}$$

The Einstein equations of motion (3.2) give for our ansatz

$$
\begin{aligned}
B'' + B'\left(B' - A' + (d-2)C'\right) &= \chi, \\
-(B'' + B'^2 - A'B') - (d-2)\left(C'' + C'^2 - A'C'\right) &= \frac{1}{2}\mathcal{M}_{ij}\partial_t\phi^i\partial_t\phi^j - \chi, \\
C'' + C'\left(-A' + B' + (d-2)C'\right) + ke^{2A-2C}(d-3) &= \chi + \frac{\mathcal{F}}{2}e^{-2B},
\end{aligned} \tag{3.7}
$$

where

$$
\chi = \frac{1}{2(d-2)}\left(-(d-3)e^{-2B}\mathcal{F} + 2l^2e^{2A}\mathcal{P}\right) \tag{3.8}
$$

and $\mathcal{F} = G_{IJ}F^I_{tx}F^J_{tx}$. The equations of motion simplify with the following relations

$$
\begin{aligned}
e^{2B} &= fe^{-2(d-3)V}, \quad e^{2A} = \frac{e^{2V}}{f}, \quad e^{2C} = t^2e^{2V}, \\
f &= -k + \frac{\mu}{t^{d-3}} - l^2t^2e^{2(d-2)V},
\end{aligned} \tag{3.9}
$$

and one obtains from (3.7) that the scalar fields must satisfy

$$
\frac{1}{2}\mathcal{M}_{ij}\partial_t\phi^i\partial_t\phi^j = -(d-2)\left[\frac{1}{t}V'(d-2) + V'^2(d-3) + V''\right]. \tag{3.10}
$$

We also obtain from (3.7) the following relation for the gauge fields

$$
\mathcal{F} = -2(d-2)e^{-2(d-3)V}\left[k\left(V'' + \frac{V'}{t}(d-2)\right) - \frac{\mu}{t^{d-3}}\left(V'' + \frac{1}{t}V'\right)\right]. \tag{3.11}
$$

Note that the l^2 drops out in the expression for the gauge fields and that the scalar fields are independent of the parameters μ and k.

As a special case, we consider theories with trivial scalars, i.e., Einstein Maxwell theory with a potential (negative cosmological constant) $\mathcal{P} = -(d-2)(d-3)$. In these cases, we obtain from (3.10)

$$
V'' + \frac{V'}{t}(d-2) + V'^2(d-3) = 0. \tag{3.12}
$$

The above equation can be solved by:

$$
e^V = \left(1 + \frac{q}{t^{d-3}}\right)^{\frac{1}{d-3}}. \tag{3.13}
$$

When substituted in (3.11), this solution gives

$$\mathcal{F} = 2(d-2)\,(d-3)\,q\frac{(kq+\mu)}{t^{2d-4}\left(1+\frac{q}{t^{d-3}}\right)^4}. \tag{3.14}$$

Note that reversing the sign of l^2 gives solutions in d-dimensional de Sitter Einstein Maxwell theories. These general solutions were considered in [22].

For theories with non-trivial scalars, one has to solve a more complicated set of equations. A natural ansatz is to express V and the gauge fields in terms of harmonic functions. Equation (3.10) can then be used to determine the scalar fields in terms of the harmonic functions. However for a general potential the scalar fields will not solve the other equations of motion. In what follows we will analyze special cases of gauged supergravity theories in which the harmonic ansatz works.

4 Symmetric potentials

In this section we consider solutions of a gauged supergravity where the scalars parameterize the subspace $SL(N,R)/SO(N,R)$ of the coset manifold of maximal gauged supergravity [2]. The Lagrangian in dimension d is given by

$$e^{-1}\,\mathcal{L}_d = R - \frac{1}{2}(\partial\vec{\varphi})^2 - \frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{\mu\nu\,J} - l^2\mathcal{P}\,, \tag{4.1}$$

where the potential $\mathcal{P}$ is a symmetric potential given by

$$\mathcal{P} = -\frac{(d-3)^2}{8}\left((\sum_{I=1}^{N}X_I)^2 - 2\sum_{I=1}^{N}X_I^2\right)\,. \tag{4.2}$$

We consider theories with gauge kinetic term given by

$$G_{IJ} = \frac{1}{(X_I)^2}\delta_{IJ}. \tag{4.3}$$

The N scalars X_I are subject to the constraint

$$\prod_{I=1}^{N}X_I = 1\,. \tag{4.4}$$

[2]Domain wall solutions in these theories were discussed in [29].

The constrained scalars X_I are parameterized in terms of $(N-1)$ independent dilatonic scalars $\vec{\varphi}$ as follows

$$X_I = e^{-\frac{1}{2}\vec{b}_I\cdot\vec{\varphi}}\,, \tag{4.5}$$

where the $\vec{b}_I$ are the weight vectors of the fundamental representation of $SL(N,R)$, satisfying

$$\vec{b}_I\cdot\vec{b}_J = 8\delta_{IJ} - \frac{8}{N}\,, \qquad \sum_I \vec{b}_I = 0\,, \qquad (\vec{u}\cdot\vec{b}_I)\,\vec{b}_I = 8\vec{u}\,. \tag{4.6}$$

The vector $\vec{u}$ is an arbitrary N-vector. The above relations allow $\vec{\varphi}$ to be determined in terms of X_I

$$\vec{\varphi} = -\frac{1}{4}\sum_I \vec{b}_I\,\log X_I\,. \tag{4.7}$$

We take the same ansatz for the metric as in the previous section

$$ds^2 = fe^{-2(d-3)V}dx^2 + e^{2V}\left(-\frac{dt^2}{f} + t^2 ds^2_{d-2,k}\right). \tag{4.8}$$

We find that the Einstein equations of motion admit solutions given by

$$\begin{aligned}
e^{2V} &= \prod (H_I)^{\frac{1}{2(d-2)}}\,, \qquad f = -k + \frac{\mu}{t^{d-3}} - l^2t^2e^{2(d-2)V}, \\
X_I &= \frac{1}{H_I}\prod_J (H_J)^{\frac{1}{4}\left(\frac{d-3}{d-2}\right)} = \frac{1}{H_I}e^{(d-3)V}, \\
F^I_{xt} &= \frac{(d-3)}{H_I^2 t^{d-2}}\sqrt{(kq_I^2 + \mu q_I)}, \\
H_I &= 1 + \frac{q_I}{t^{d-3}}.
\end{aligned} \tag{4.9}$$

provided the following relation holds

$$\vec{b}_I.\vec{b}_J = 8\delta_{IJ} - 2\frac{d-3}{d-2}. \tag{4.10}$$

It follows from the first equation in (4.6) that $N = 4(d-2)/(d-3)$. Since N and d have to be integers, this relation holds for $d = 4, 5, 7$, where $N = 8, 6, 5$ respectively.

We now turn to the scalar equation of motion which for our ansatz reads

$$-\frac{1}{4}\partial_{\vec{\phi}}G_{JK}F^J_{\mu\nu}F^{K\mu\nu} + \frac{1}{\sqrt{g}}\partial_t(\sqrt{g}g^{tt}\partial_t\vec{\phi}) - l^2\partial_{\vec{\phi}}\mathcal{P} = 0. \tag{4.11}$$

Using the fact that $\sum_I \vec{b}_I = 0$, as well as $\frac{\partial X_I}{\partial\vec{\phi}} = -\frac{1}{2}\vec{b}_I X_I$, it is easy to show that the scalar equation of motion is satisfied for the scalar potential given in (4.2).

5 Five dimensional $N = 2$ gauged supergravity

Here we will consider the five dimensional supergravity theory obtained from dimensionally reducing type-IIB supergravity on S^5. This theory has three abelian vector multiplets (including the graviphoton). The scalars $X_I = S, T, U$ of this theory satisfy the constraint $X_1 X_2 X_3 = 1$ and thus the theory has two independent scalar fields. Taking X_1, X_2 as the independent variables, the potential for this theory is given by

$$\mathcal{P} = -4\Big(\frac{1}{X_1} + \frac{1}{X_2} + X_1 X_2\Big). \tag{5.1}$$

The action can be written in the form (3.1) with

$$\mathcal{M}_{ij} = \begin{pmatrix} \frac{2}{X_1^2} & \frac{1}{X_1 X_2} \\ \frac{1}{X_1 X_2} & \frac{2}{X_2^2} \end{pmatrix}, \qquad G_{IJ} = \begin{pmatrix} \frac{1}{X_1^2} & 0 & 0 \\ 0 & \frac{1}{X_2^2} & 0 \\ 0 & 0 & X_1^2 X_2^2 \end{pmatrix}.$$

Here we have ignored a Chern-Simons term which is not relevant for our solutions. Note that if we write $X_I = e^{-\frac{1}{2}(\vec{a}_I.\vec{\varphi})}$ with

$$\vec{a}_1 = \left(\frac{2}{\sqrt{6}}, \sqrt{2}\right), \quad \vec{a}_2 = \left(\frac{2}{\sqrt{6}}, -\sqrt{2}\right), \quad \vec{a}_3 = \left(-\frac{4}{\sqrt{6}}, 0\right), \tag{5.2}$$

then the kinetic term for the scalar fields takes the canonical form $-\frac{1}{2}\left(\partial\vec{\phi}\right)^2$.

The ansatz for S-brane metric is

$$ds^2 = e^{-4V(t)} f(t) dx^2 - \frac{e^{2V}}{f(t)} dt^2 + e^{2V} t^2 (d\theta^2 + \sinh^2\theta d\Omega_2^2), \tag{5.3}$$

where e^{6V} and f are given by

$$e^{6V} = h_1(t) h_2(t) h_3(t), \qquad f(t) = 1 + \frac{\mu}{t^2} - l^2 t^2 e^{6V}. \tag{5.4}$$

and h_I , $I = 1, 2, 3$, is a harmonic function given by

$$h_I(t) = 1 + \frac{q_I}{t^2}. \tag{5.5}$$

The scalars and the gauge field-strength can be expressed as

$$X_I = \frac{e^{2V(t)}}{h_I(t)}, \quad F^I_{tx} = \frac{2}{h_I(t)^2} \frac{\tilde{q}_J}{t^3}. \tag{5.6}$$

All equations of motion will be satisfied if the following relation between $q_I, \tilde{q}_i$ holds

$$(\tilde{q}_I)^2 = -q_I^2 + \mu q_I, \tag{5.7}$$

which can be represented as

$$q_I = \mu \sin^2 \beta_I, \quad \tilde{q}_I = \mu \sin \beta_I \cos \beta_I. \tag{5.8}$$

Note that (5.7) implies that it is impossible to have an extremal solution (where $\mu = 0$) with real $\tilde{q}_I$, i.e. real gauge fields.

The zeros of the function $f(t)$ given in (5.4) determine the location of the horizon. Continuation past the horizon produces a static metric, which can be identified with a topological black hole solution of the gauged supergravity. This is similar to the case in the simple five dimensional AdS black hole discussed in section 2.

Setting the coupling $l = 0$ in the action gives an ungauged $N = 2$ supergravity. The S-brane solution we have found in the gauged supergravity is also a solution in this limit, giving an S-brane in asymptotically flat space. This solution generalizes the S-brane related to the RN black hole found in [1, 10, 11]. The time dependent metric (5.3) has a horizon at $t = 0$. Continuing the coordinates past the horizon produces a static spacetime with a timelike curvature singularity. The causal structure of the spacetime is very similar to the metric (2.3).

The solution can be related by analytical continuation to the black hole solution found in [30]

$$\tau \to ir, \quad x \to it, \quad \tilde{q}_I \to i\tilde{q}_I, \quad \theta \to i\theta. \tag{5.9}$$

The continued charges satisfy the condition

$$\tilde{q}_I^2 = q_I^2 + \mu q_I \tag{5.10}$$

which can be solved by

$$q_I = \mu \sinh^2 \beta_I, \quad \tilde{q}_I = \mu \sinh \beta_I \cosh \beta_I. \tag{5.11}$$

Note that for the black hole solutions, it is possible to obtain extremal solutions by setting $\mu = 0$ while maintaining the reality of the gauge field.

6 A gauged supergravity in four dimensions

The Kaluza Klein reduction of eleven dimensional supergravity on S^7 gives rise to $N = 8$, $d = 4$ gauged supergravity with gauge group $SO(8)$. There exists an abelian truncation with four gauge fields A_I, and four scalars X_I satisfying the constraint $X_1X_2X_3X_4 = 1$ [31–33]. If we choose the independent scalars to be X_1, X_2 and X_3, the potential of the gauged theory will take the form

$$\mathcal{P} = -\left(X_1X_2 + X_1X_3 + X_2X_3 + \frac{1}{X_1X_2} + \frac{1}{X_1X_3} + \frac{1}{X_2X_3} \right). \tag{6.1}$$

The metrics $\mathcal{M}_{ij}$ and G_{IJ} are given by

$$\mathcal{M}_{ij} = \begin{pmatrix} \frac{2}{X_1^2} & \frac{1}{X_1X_2} & \frac{1}{X_1X_3} \\ \frac{1}{X_1X_2} & \frac{2}{X_2^2} & \frac{1}{X_2X_3} \\ \frac{1}{X_1X_3} & \frac{1}{X_2X_3} & \frac{2}{X_3^2} \end{pmatrix}, \quad G_{IJ} = \begin{pmatrix} \frac{1}{X_1^2} & 0 & 0 & 0 \\ 0 & \frac{1}{X_2^2} & 0 & 0 \\ 0 & 0 & \frac{1}{X_3^2} & 0 \\ 0 & 0 & 0 & X_1^2X_2^2X_3^2 \end{pmatrix}. \tag{6.2}$$

Note that if we write $X_I = e^{-\frac{1}{2}(\vec{a}_I.\vec{\phi})}$ with

$$\vec{a}_1 = (1,1,1)\,, \quad \vec{a}_2 = (1,-1,-1)\ ,\vec{a}_3 = (-1,1,-1)\,, \quad \vec{a}_4 = (-1,-1,1)\,,$$

then the kinetic term of the scalar fields takes the canonical form.

The ansatz for the S-brane metric is

$$ds^2 = e^{-2V(t)} f(t) dx^2 + e^{2V} \left(-\frac{1}{f(t)} dt^2 + t^2(d\theta^2 + \sinh^2\theta d\phi^2) \right), \tag{6.3}$$

where

$$f = 1 + \frac{\mu}{t} - l^2t^2e^{4V}, \quad e^{4V(t)} = h_1(t)h_2(t)h_3(t)h_4(t). \tag{6.4}$$

h_I is a harmonic function given by

$$h_I(t) = 1 + \frac{q_I}{t}. \tag{6.5}$$

The scalars and the gauge field-strength can be expressed as

$$X_I = \frac{e^V}{h_I}, \quad F^I_{tx} = \frac{\tilde{q}_I}{h_I^2(t)t^2}. \tag{6.6}$$

The equations of motion will be satisfied if the following relation holds

$$(\tilde{q}_I)^2 = -q_I^2 + \mu q_I. \tag{6.7}$$

7 A gauged supergravity in seven dimension

The Kaluza Klein reduction of eleven dimensional supergravity on S^4 gives rise to a $d = 7$ gauged supergravity with gauge group $SO(5)$ [34,35]. It is possible to truncate the theory to an abelian $U(1)^2$ subgroup, with two gauge fields $A_I, I = 1, 2$ and two scalars X_i, $i = 1, 2$.

The metrics $\mathcal{M}_{ij}$ and G_{IJ} and the potential are given by

$$\mathcal{M}_{ij} = \begin{pmatrix} \frac{3}{X_1^2} & \frac{2}{X_1 X_2} \\ \frac{2}{X_1 X_2} & \frac{3}{X_2^2} \end{pmatrix}, \quad G_{IJ} = \begin{pmatrix} \frac{1}{X_1^2} & 0 \\ 0 & \frac{1}{X_2^2} \end{pmatrix},$$
$$\mathcal{P} = -16 X_1 X_2 - 8 \left(\frac{1}{X_1 X_2^2} + \frac{1}{X_2 X_1^2} \right) + \frac{2}{(X_1 X_2)^4}. \tag{7.1}$$

Like in four and five dimensions, the kinetic term for the scalar fields take the canonical form if we write $X_i = e^{-\frac{1}{2}(\vec{a}_i.\vec{\phi})}$ with $\vec{a}_1 = \left(\sqrt{2}, \sqrt{\frac{2}{5}}\right)$, $\vec{a}_2 = \left(-\sqrt{2}, \sqrt{\frac{2}{5}}\right)$.

The ansatz for the S-brane solution is

$$ds^2 = e^{-8V(t)} f(t) dx^2 + e^{2V} \left(-\frac{1}{f(t)} dt^2 + t^2 (d\theta^2 + \sinh^2 \theta d\Omega_4^2) \right),$$
$$f(t) = 1 + \frac{\mu}{t^4} - l^2 t^2 e^{10V}, \quad e^{10V(t)} = (1 + \frac{q_1}{t^4})(1 + \frac{q_2}{t^4}),$$
$$X_1 = e^{4V} (1 + \frac{q_1}{t^4})^{-1}, \quad X_2 = e^{4V} (1 + \frac{q_2}{t^4})^{-1},$$
$$F^1_{tx} = 4 \frac{\tilde{q}_1}{t^5} (1 + \frac{q_1}{t^4})^{-2}, \quad F^2_{tx} = 4 \frac{\tilde{q}_2}{t^5} (1 + \frac{q_2}{t^4})^{-2}. \tag{7.2}$$

The equations of motion will be satisfied if the following relation holds

$$(\tilde{q}_I)^2 = -q_I^2 + \mu q_I, \qquad I = 1, 2. \tag{7.3}$$

Note that the condition on the charges in this and the previous section is exactly the same as in five dimensions (5.10). The solutions share the same structure and properties: For example, an analytic continuation (5.9) relates the S-brane solutions to the black hole solutions found in [31].

8 Discussion

In this paper we have considered S-brane solutions of gauged and ungauged supergravity theories in various dimensions. The solutions have a close relationship with black holes

and can be obtained by an analytic continuation from static black hole solutions. This analytic continuation modifies the charge parameters and the relation (5.7) implies that, for the S-brane solutions, there exists no 'non-extremal' limit which keeps the gauge fields in the solution real. This is in agreement with the expectation that S-brane solutions cannot be supersymmetric since they do not posses timelike or null Killing vectors. [3]

In the bosonic action of the gauged supergravities the only term which is produced by the gauging is the scalar potential. In the limit $l \to 0$ the gauged supergravity goes over to an ungauged supergravity. In this limit the solutions which we presented go over to S-branes in asymptotically flat space. They generalize known S-brane solutions [9] since they have both nontrivial gauge and scalar fields. The solutions have a horizon at $t = 0$ which separates the time dependent S-brane spacetime from a static spacetime. In the static part of the spacetime there is a timelike curvature singularity. In [9] the timelike singularity in the S-brane spacetime was interpreted as a negative tension object. The supergravities we have considered have a natural interpretation as compactifications of string theory or M theory. It would be interesting to check whether the singularity can be identified with an orientifold plane in the full string theory. It might also be interesting to investigate the possibility of nonsingular S-brane solutions by constructing di-hole solutions [38] in the supergravity theories we have considered and analytically continuing them along the lines of [18].

The apparent similarities for the three S-brane solutions we have presented for the gauged supergravities in $d = 4, 5, 7$ lie in the fact that they can be obtained from the general solution of the gauged supergravity with symmetric potentials presented in section 4. This can be done by suitable identification of gauge and scalar fields. Such an identification has been discussed in [29] in the study of domain wall solutions. It would be very interesting to generalize the solutions for the symmetric scalar potential to more general gauged supergravity theories. It is expected that the simple harmonic ansatz will not work in the general cases and that the solutions will be more complicated. It would be interesting to investigate this further and in particular to find a criterion for when a harmonic ansatz will work.

[3]However, we note that upon the replacement of l by il, our solutions become relevant to the theories of de Sitter supergravity [36] which were obtained from the reduction of IIB* and M* theory [37]. Note also that in these models, time dependent 'extremal' solutions are possible.

The S-brane solutions in gauged supergravities we have found possess the property that continuation beyond the horizon relates them to topological black hole solutions in AdS. In this respect it might be interesting to study the AdS S-brane solutions from the point of view of a dual CFT. It would also be interesting to lift the solutions to ten and eleven dimensions [31] and study their properties. We leave these questions for future work.

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